MIL-STD-781C AND CONFIDENCE INTERVALS ON MEAN TIME BETWEEN FAILURES

AES-7905

FOUNDED IN 1795
UNION COLLEGE
SCHENECTADY, NY

ADMINISTRATIVE AND ENGINEERING SYSTEMS MONOGRAPH

(14) AES-79Ø5

(6)

MIL-STD-781C

AND

CONFIDENCE INTERVALS ON MEAN TIME BETWEEN FAILURES

by

(10) Josef Schmee
Institute of Administration and Management
Union College

(12) 48p.

(11) 28 Febr[redacted]79

(15) N00014-77-C-0438

(9) Technical rept.
1 Jun 78-1 Mar 79

UNION COLLEGE AND UNIVERSITY
INSTITUTE OF ADMINISTRATION AND MANAGEMENT
Schenectady, New York 12308

408 725

LB

ACKNOWLEDGMENT

The author appreciates the substantial criticism and administrative support of Professor L.A. Aroian, Union College, and Dr. Gerald J. Hahn, General Electric Corporate Research and Development.

MIL-STD-781 and Confidence Intervals

SUMMARY

Various realistic examples illustrate how to obtain confidence limits on the mean time between failures (MTBF) of an exponential distribution from data obtained from one of the fixed-size or sequential test plans of MIL-STD-781C.

For fixed-length tests, the methods developed by B. Epstein and the modifications of H.L. Harter are briefly discussed. For the sequential tests simple charts for newly developed methods of Bryant and Schmee are given.

INTRODUCTION

MIL-STD-781C "covers the requirements for reliability qualification tests and reliability acceptance tests for equipment that experiences a distribution of times-to-failure that is exponential" [12]. A set of standard test plans are provided. They are either of the fixed length or the sequential type. The performance requirement is specified in terms of mean-time-between-failure (MTBF). Sometimes dimensions other than time are used, e.g. cycles. Then the performance requirement is mean-cycles-between-failures. MIL-STD-781C is only applicable when the times to failure follow the exponential distribution.

One of the major criticisms of a previous version of the standard (MIL-STD-781B) was that equipment tested and accepted by the statistical test plans often showed unacceptable time to failure characteristics in the field. Such discrepancies between a test method and the field may be due to statistical and non-statistical reasons.

The test plans in MIL-STD-781B emphasized statistical hypothesis testing of two distinct values of the MTBF, θ_o versus θ_1. Either θ_o was accepted and θ_1 rejected, or vice versa. The accepted value was assumed to be the MTBF of the tested equipment. However, acceptance or rejection of a statistical hypothesis provides only limited insight into the possible values of the MTBF. On the other hand, a confidence interval calculated from the test data after acceptance or rejection of the equipment, provides a range of values of statistical hypotheses (or MTBFs) which could not be rejected on the basis of the test data. Thus a confidence interval is viewed as a collection of acceptable hypotheses. Confidence intervals are new in MIL-STD-781C [12].

As a specific example, in a later section we calculate a confidence interval on the MTBF of some electronic equipment from 80 hours to 241 hours. This means that <u>any</u> hypothesis that the MTBF is between 80 hours and 241 hours would have been accepted, and not merely the 100 hours as stated in the accepted hypothesis of that example. Rather than accepting (or rejecting) a single value for the MTBF, with a confidence interval one can give a range of values for which a similar decision would have been reached. This is useful to know, because in MIL-STD-781 (and in other real world situations) the acceptance or rejection of the statistical hypothesis is frequently accompanied by a contractual acceptance or rejection of equipment.

This paper presents an overview of classical methods for confidence intervals on the MTBF of an exponential distribution after completion of a life test of MIL-STD-781C. The methods themselves are not limited to the standard, butapply (especially after fixed-length tests) after testing assuming an exponential distribution. The next section briefly reviews the test plans of MIL-STD-781C. This is followed by sections on confidence intervals after fixed-length tests and after the sequential test plans.

The following are limits to the subject treated in this paper.

- Only the statistical aspects of the test plans are considered. Thus the important problem of lab versus field testing is not considered (see Yasuda [15]).
- Only equipment with failure times that are either exponential or can be transformed to the exponential can be considered. Harter and Moore [10] looked at the robustness of the test plans if the assumption of exponentiality is not satisfied. In particular, they look at Weibull failure times.
- Only confidence intervals on the MTBF after a statistical test are discussed. This excludes the discussion of prediction intervals or tolerance intervals. The various types of intervals are discussed in Hahn [6,7].

  A prediction interval is an interval which contains a future outcome or outcomes with a specified probability, for example,

  - the time to failure of a single equipment, or
  - the average time to failure of the equipment in a lot of size k, or
  - all the failure times of the equipment in a lot of size k.

  Prediction intervals are generally wider than confidence intervals. Using a confidence interval when a prediction interval is required results in a wrong, overly optimistic answer. Tolerance intervals contain the failure times of a least a specified proportion p (of the population) with a stated level of confidence. Tolerance intervals are generally also wider than confidence intervals. Many times rather than confidence intervals, prediction intervals or tolerance intervals are the answer. New methods have yet to be worked out for these types of intervals.

STATISTICAL TEST PLANS

The test plans of MIL-STD-781C serve two major purposes. In (preproduction) qualification tests they are used to ensure that hardware reliability meets or exceeds the requirements. Also they are used to conduct (production) acceptance tests either through lot-by-lot sampling or for all equipment.

This section introduces the standard test plans. First, notation and definitions are given. Then fixed-length tests and sequential tests are briefly described and compared.

Notation:

$f(t)$	=	$(1/\Theta) \exp\{-t/\Theta\}$, $t>0$; the density function of exponential failure times.
Θ	=	the true mean time between failures (MTBF) of the exponential distribution.
Θ_1	=	lower test MTBF is an unacceptable value of the MTBF which the standard test plans reject with high probability.
Θ_0	=	upper test MTBF is an acceptable value of MTBF equal to the discrimination ratio times Θ_1.
d	=	Θ_0/Θ_1, the discrimination ratio; d identifies a test plan.
α	=	producer's risk; the probability of rejecting equipment(s) with a true MTBF equal to Θ_0.

β = consumer's risk, the probability of accepting equipment(s) with the true MTBF equal to Θ_1.

t_{Ai} = standardized acceptance time; equipment is accepted, if not more then i failures occur in $t_{Ai}\Theta_1$ hours.

t_{Ri} = standardized rejection time; equipment is rejected, if at least i failures occur at or before $t_{Ri}\Theta_1$ hours.

$\overline{\underline{\Theta}}$ = demonstrated MTBF; as defined in the standard it is the probable range of the true MTBF stated with a specified degree of confidence. In this paper $\underline{\Theta}<\Theta<\overline{\Theta}$ is the notation used for confidence intervals.

$\hat{\Theta}$ = t/r = total test time t/number of failures r; a point estimate of Θ. (Note: This is the maximum likelihood estimate for both fixed-length and sequential test plans.)

Standard Test Plans: The standard test plans of MIL-STD-781C provide for various combinations of producer's risks (α), consumer's risks (β), and discrimination ratios (d). These three parameters identify a particular test plan. The plans can be separated into three groups:

1. Fixed-length test plans, numbered IXC through XVIIC, and XIXC through XXIC.
2. Probability ratio sequential tests (PRST), numbered IC through VIIIC.

3. All equipment reliability test, number XVIIIC (not covered in this paper).

Parameters of the Test Plans: The test plans in the above first two groups are characterized by the way a test is eventually terminated (stopping rule, truncation), and, most important, by the three parameters α, β, and d. The decision risks α and β of the standard test plans are .1, .2, or .3; the discrimination ratio is either 1.5, 2.0, or 3.0.

For example, to test the statistical hypotheses

$$H_o: \quad \Theta_o = 10 \text{ hours versus}$$
$$H_1: \quad \Theta_1 = 5 \text{ hours}$$

i.e. d = 2.0 with specified risks $\alpha = \beta = .1$, one can either select the fixed-length test XIIC or the sequential test IIIC. Table C-I of MIL-STD-781C (12, p. 64), gives a summary of the parameters of each test plan.

The same test plan would be chosen for testing

$$H_o: \quad \Theta_o = 30 \text{ hours versus}$$
$$H_1: \quad \Theta_1 = 15 \text{ hours,}$$

since the discrimination ratio d = 30/15 = 10/5 = 2 is the same, assuming the same decision risks. However, the different hypotheses make a difference, because the times to rejection and times to acceptance are multiples of Θ_1. Thus for acceptance in test plan XIIC, the second hypotheses requires three times the total test time of the first hypothesis, viz 15 x 18.8 hours as opposed to 5 x 18.8 hours. In fixed-length tests the minimum time to accept is always a multiple of Θ_1. The standard minimum times to accept are also given in Table C-I of MIL-STD-781C.

In sequential test plans the standard acceptance times t_{Ai} and the standard rejection times t_{Ri} must be multiplied by Θ_1 to arrive at the actual acceptance and rejection times. For illustration, standard acceptance and rejection times for test plan IIIC are given in Table 1, for the other sequential test plans they are in MIL-STD-781C (12, pp. 66-81). For example, for test plan IIIC t_{A0} = 4.40, t_{A1} = 5.79 and so on. Thus, the first (second) hypothesis can be accepted, if either

- no failure occurs up to $t_{A0}\Theta_1$ = 4.40 x 5 hours (4.40 x 15 hours), or
- one failure occurs before $t_{A0}\Theta_1$, and no failure occurs between 4.40 x 5 hours (4.40 x 15 hours), and

  t_{A1} = 5.79 x 5 hours (5.79 x 15 hours), and so on.

<u>Nominal versus True Decision Risks</u>: The nominal decision risks are used to identify comparable test plans. Because failures are measured by whole numbers, it is generally not possible to construct a test with stated risks. The risks actually achieved are called <u>true decision risks</u>. They are very close to the nominal risks.

For example, for test plan XIIC the nominal risks are $\alpha = \beta = 0.10$, but the true risks are $\alpha' = 0.096$ and $\beta' = 0.106$. The true decision risks for the other test plans are given in Tables II-V of MIL-STD-781C (12, pp. 12-3).

Selection of a Test Plan: One must choose between fixed-length or sequential tests. The standard explains that a fixed-length test must be chosen if

- the total test time is fixed in advance, or
- an estimate of the true MTBF demonstrated is required.

Sequential tests are recommended when only an accept/reject decision is desired.

These preceding selection criteria seem rather arbitrary because the maximum total test time (truncation time) of a sequential test is hardly longer than the fixed-length minimum acceptance time. For example, the truncation time for test plan IIIC is 20.6 θ_1 hours, whereas the minimum acceptance time for the equivalent fixed-length test plan XIIC is 18.8 θ_1 hours, at worst an increase of 1.8 θ_1 hours or 8.6 percent. However, sequential tests offer substantially earlier termination times. Test plan IIIC terminates on the average after 10.2 θ_1 hours.

Bryant and Schmee [5] and the graphs of this paper provide equivalent methods to those available for fixed-length tests for estimation after a sequential test.

Sample Size and Test Length: The standard also specifies a minimum sample size for production reliability acceptance of at least three equipments (unless otherwise specified), or between 10% and 20% of the lot. The sample size for a reliability qualification test is specified in the contract. Also, each equipment shall operate at least one-half the average operating time of all equipment on test.

ESTIMATION AFTER A FIXED-LENGTH TEST

In estimation from life test data one must distinguish between time censored data, when the test is terminated after some predetermined time, and failure censored data, when the test is terminated at the occurrence of a predetermined number of failures. Each censoring mode requires different formulae. In life tests, such as those of MIL-STD-781, either censoring mode may occur: time censoring occurs, if the test is accepted; failure censoring, if the test is rejected. However, at the start of the test one does not know, which of the two censoring modes will occur, so that a set of formulae or tables fitting each outcome must be specified.

MIL-STD-781C provides methods for estimation after a fixed-length test (but not after a sequential test). In this section two methods for estimation after a fixed-length test are presented. The first, due to Epstein [4], is the one currently included in MIL-STD-781C. It yields confidence intervals with higher confidence levels than stated. The second method, proposed by Harter [9], seems to give narrower intervals at confidence levels closer to the stated ones than Epstein's method. Because of the form of the exponential distribution both methods do not require the actual failure times. Only the number of failures and the total test time are accumulated. The same holds for the methods after sequential tests described in the next section.

<u>Epstein's Method</u>: Epstein[4] proposes the following formulae for two-sided $(1-2\gamma)$ 100% confidence intervals on the MTBF after a fixed-length test:

After Acceptance:

$$\underline{\Theta} = \frac{2t}{\chi^2(1-\gamma,\ 2r+2)} < \Theta < \frac{2t}{\chi^2(\gamma,\ 2r)} = \underline{\Theta} \text{ if } r > 0$$

and

$$\underline{\Theta} = \frac{2t}{\chi^2(1-\gamma,\ 2)} < \Theta < \infty \quad \text{if } r = 0$$

After Rejection:

$$\underline{\Theta} = \frac{2t}{\chi^2(1-\gamma,\ 2r)} < \Theta < \frac{2t}{\chi^2(\gamma,\ 2r)} = \overline{\Theta}$$

where

t = total test time when the life test is stopped,

r = number of accumulated failures when the life test is stopped,

$\chi^2(\gamma, f) = \gamma(100)$ percentile of the χ^2-distribution with f degrees of freedom.

Remarks:

1. The percentiles of the χ^2-distribution are given for 40%, 60% and 80% two-sided confidence intervals in Table VI of (12, p. 19) or in many standard statistical text books. More complete tabulations

are given in Harter [8].

2. For $(1-\gamma)$ 100% one-sided confidence intervals one uses the same formulae as for $(1-2\gamma)$ 100% two-sided confidence intervals. For one-sided lower intervals the left-hand side of the two-sided formula is used (the upper limit is at infinity), and for one-sided upper intervals the right-hand side of the two-sided formula is used (the lower limit is zero). Also note that there is no one-sided upper confidence interval with zero failures $(r=0)$.

   MIL-STD-781C does not even give the formula for $r=0$ for two-sided confidence intervals. The obvious reason for this omission is that this results in an interval which is unbounded to the right.

3. The above formulae produce conservative confidence intervals. This means that the true confidence level is usually higher than stated (see Epstein [4] and Fairbanks [5]).

Example : In a fixed-length life test of electronic equipment it is desired to accept the equipment with probability $1-\alpha = .9$ when $\Theta=\Theta_0 = 100$ hours, and to reject it with probability $1-\beta = .9$ when $\Theta=\Theta_1 = 50$ hours. Thus the discrimination ratio $d = 2.0$. Test Plan XIIC is selected for this test.

From Table II of MIL-STD-781C (12, p. 12) we find that this test plan results in acceptance, if not more than 13 failures occur in $18.8\,\Theta_1 = 18.8 \times 50 = 940$ hours, and in a rejection otherwise.

Acceptance: Suppose that only $r = 7$ failures occur in 940 hours. So the test results in acceptance of the equipment. In this case the data are time censored. Note that the seventh failure occurred before 940 hours.

A two-sided 80% confidence interval on the MTBF is

$$\underline{\Theta} = \frac{2 \times 940}{\chi^2(.90,\ 16)} < \Theta < \frac{2 \times 940}{\chi^2(.10,\ 14)} = \overline{\Theta}$$

$$\underline{\Theta} = \frac{1880}{23.5418} = 79.86 < \Theta < \frac{1880}{7.7895} = 241.35 = \overline{\Theta}$$

This means that the true MTBF is, with 80% confidence, longer than 80 hours and shorter than 241 hours. The upper test MTBF $\Theta_0 = 100$ hours is included in this interval, the lower test MTBF $\Theta_1 = 50$ hours is not.

Rejection: In a life test of another lot of the above equipment, the 14-th failure occurs after 850 hours. The test results in rejection of the equipment and is terminated before the full length of 940 hours. Thus the data are failure censored.

A two-sided 80% confidence interval on the MTBF is

$$\frac{2 \times 850}{\chi^2(.90,\ 28)} < \Theta < \frac{2 \times 850}{\chi^2(.10,\ 28)}$$

$$\underline{\Theta} = \frac{1700}{37.9159} = 44.84 < \Theta < \frac{1700}{18.9392} = 89.76 = \overline{\Theta}$$

This means that the true MTBF is, with 80% confidence, longer than 45 hours and shorter than 90 hours. The upper test MTBF Θ_0 = 100 hours is not included in this interval, however the lower test MTBF Θ_1 = 50 hours is included.

Harter's Method: Harter [9] replaces Epstein's "after acceptance" formulae by a heuristic formula due to D.R. Cox, and continues to use Epstein's "after rejection" formula. By Monte Carlo simulation he shows that this combination results in confidence intervals with confidence levels closer to the stated ones than Epstein's method. Harter proposes the following formulae for two-sided $(1-2\gamma)$ 100% confidence intervals on the MTBF after a fixed-length test.

After Acceptance:

$$\underline{\Theta} = \frac{2t}{\chi^2(1-\gamma,\ 2r+1)} < \Theta < \frac{2t}{\chi^2(\gamma,\ 2r+1)}$$

After Rejection (same as Epstein's Method):

$$\underline{\theta} = \frac{2t}{\chi^2(1-\gamma,\ 2r)} < \theta < \frac{2t}{\chi^2(\gamma,\ 2r)} = \overline{\theta}$$

The notation is the same as before.

Remarks:

1. After acceptance Harter's method yields shorter intervals than Epstein's. The relative difference decreases as the number of failures increases. The true confidence levels are on the average closer to the stated confidence levels for Harter's method than for Epstein's.
2. For zero failures (r=0), Harter's method yields bounded two-sided confidence intervals, Epstein's does not.
3. For $(1-\gamma)$ 100% one-sided confidence intervals one uses the same formulae as for $(1-2\gamma)$ 100% two-sided confidence intervals. For a one-sided lower intervals the left-hand side of the two-sided formula is used (the upper limit is at infinity), and for one-sided upper intervals the right-hand side of the two-sided formula is used (the lower limit is zero).
4. The intervals are heuristic with limited theoretical justification. However, they work very well.

Example: This is the same example as given under Epstein's method.

Acceptance: Suppose that only r=7 failures occur in 940 hours.

A two-sided 80% confidence interval on the MTBF is

$$\underline{\Theta} = \frac{2 \times 940}{\chi^2(.90,\ 15)} < \Theta < \frac{2 \times 940}{\chi^2(.10,\ 15)} = \overline{\Theta}$$

$$\underline{\Theta} = \frac{1880}{22.3072} = 84.28 < \Theta < \frac{1880}{8.5468} = 219.97 = \overline{\Theta}$$

This means that the true MTBF is, with 80% confidence, larger than 84 hours and smaller than 220 hours. This compares to 80 hours and 241 hours for Epstein's confidence intervals. In this example Harter's interval is 30 hours shorter than Epstein's.

Rejection: Suppose that the 14-th failure occurs at 850 hours. Then using the same calculations as for Epstein's method, the two-sided 80% confidence interval is from 44.84 hours to 89.76 hours.

ESTIMATION AFTER A SEQUENTIAL TEST

This section presents charts for obtaining confidence intervals on the exponential MTBF after a sequential test. They are based on the work of Bryant and Schmee [3]. Previously various attempts at sequential estimation have been made by Sumerlin [14], Aroian and Oksoy [2], and Luetjen [11]. They are briefly discussed in Bryant and Schmee [3].

The use of the charts given here is similar to the formulae for estimation after a fixed-length test. There are separate charts for tests resulting in acceptance and those resulting in rejection. As with Epstein's method the associated overall confidence level is conservative. This means that the intervals hold for a confidence level at least as high as stated.

The charts are more convenient to use than the tables given in Bryant and Schmee [3]. Particularly when a test ends in rejection, the tables have to be interpolated but the charts do not. A disadvantage of the use of the charts is the limited accuracy with which the multipliers can be read.

For each test plan there are two charts, one for accept decisions and one for reject decisions. For test plans VIC and VIIIC after acceptance numerical values are given instead of the charts (Table 2). There are very few acceptance points and so charts did not seem advisable.

The charts contain lower and upper lines marked 5%, 10%, 20%, 30%. Rejection charts also contain a 50% line. Multipliers from the 10% lines can be used to find 90% one-sided (upper or lower) confidence intervals, or two-sided 80% confidence intervals. Similarly one uses the 5% (20%)(30%) lower or upper lines to construct 95% (80%)(70%) one-sided lower or upper confidence intervals, or 90% (60%)(40%) two-sided confidence intervals on the MTBF.

Example for Confidence Intervals after Acceptance:

In this example a sequential test similar to the fixed-length test example of the previous section is described. Electronic equipment is tested with the following specs: $\Theta_0 = 100$ hours, $\Theta_1 = 50$ hours, $\alpha = \beta = 0.10$, and $d = 2.0$. Test Plan IIIC is used.

In this test six relevant failures occurred after the following accumulated total test times: 56.3, 137.9, 201.3, 388.7, 501.4, 510.8 hours. The test results in acceptance after 636 hours, since during that time only six relevant failures occurred. The test could not have resulted in acceptance with five failures, since the sixth failure occurred before $t_{A5}\Theta_1 = 11.34 \times 50 = 567$ hours, nor could it have been accepted earlier, nor rejected.

To calculate 80% two-sided confidence limits one proceeds as follows (see Figure 1a):

1. Go to the acceptance chart for Test Plan IIIC and mark the number of failures (six) on the horizontal axis.
2. Go up the vertical line and mark the points of intersection with the 10% lower and 10% upper lines.

3. Draw horizontal lines through the points of intersections, and mark the point of intersection of the horizontal line with the vertical axis.
4. Read off the lower and upper multipliers from the vertical axis; the lower multiplier is 1.07, the upper multiplier is 3.36.
5. Multiply the lower (upper) multiplier by $\Theta_1 = 50$ to obtain the lower limit $\underline{\Theta}$ (upper limit $\overline{\Theta}$).

Thus the 80% two-sided confidence interval on Θ is

$$\underline{\Theta} = 1.07 \times 50 = 53.50 < \Theta < 3.36 \times 50 = 168.0 = \overline{\Theta}$$

This means that with 80% confidence the true MTBF is longer than 54 hours and shorter than 168 hours. The lower test MTBF $\Theta_1 = 50$ hours is not included in this interval, but the upper test MTBF $\Theta_o = 100$ hours is included.

Example for Confidence Intervals after Rejection:

As before we test equipment with Test Plan IIIC, and assume $\Theta_o = 100$ hours, $\Theta_1 = 50$ hours, $\alpha = \beta = 0.10$, and $d = 2.0$.

The actual relevant failure times are now recorded as 10.2, 12.7, 37.7, 108.3, 187.4, 267.2, 302.6 hours. The test results in a rejection after 302.6 hours, since the seventh failure occurs before the critical failure time $t_{R7} \times \Theta_1 = 6.24 \times 50 = 312$ hours. The test could not have been rejected after 267.2 hours with six failures since the

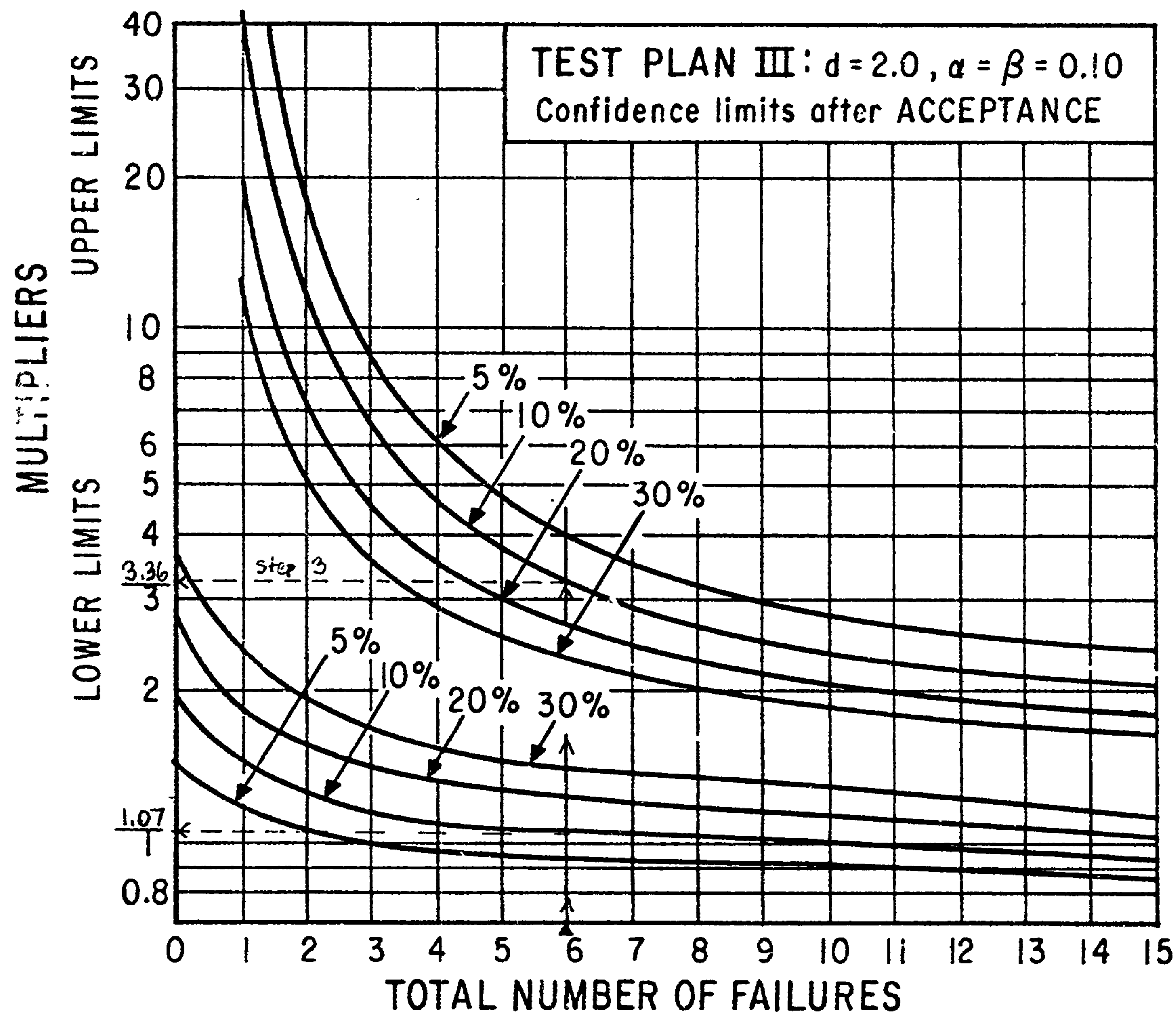


Figure 1a: Example of Steps in Obtaining 80% Confidence Limits After Acceptance

critical failure time $t_{R6} \times \Theta_1 = 4.86 \times 50 = 243$ hours is smaller than the actual failure time; nor could it have been rejected at any of the previous failures; nor could it have been accepted.

In order to calculate an 80% two-sided confidence interval one proceeds as follows (see Figure 1b):

1. Go to the appropriate chart of Test Plan IIIC after rejection and mark the standardized total test time which is equal to (total test time t)/Θ_1 = 302.6/50 = 6.05 hours.
2. Draw a vertical line and mark the points of intersection with the 10% lower and 10% upper lines.
3. Draw horizontal lines through the points of intersections, and mark the point of intersection of the horizontal line with the vertical axis.
4. Read off the lower and upper multiplier from the vertical axis; the lower multiplier is 0.58, and the upper multiplier is 1.75.
5. Multiply the lower (upper) multipliers by Θ_1 = 50 hours to obtain the lower limit $\underline{\Theta}$ (upper limit $\overline{\Theta}$).

Thus the 80% two-sided confidence interval on Θ is

$\underline{\Theta} = 0.58 \times 50 = 29.00 < \Theta < 1.75 \times 50 = 75.00 = \overline{\Theta}$

This means that with 80% confidence the true MTBF is longer than 29 hours but shorter than 75 hours. The lower test MTBF Θ_1 = 50 hours is included in this

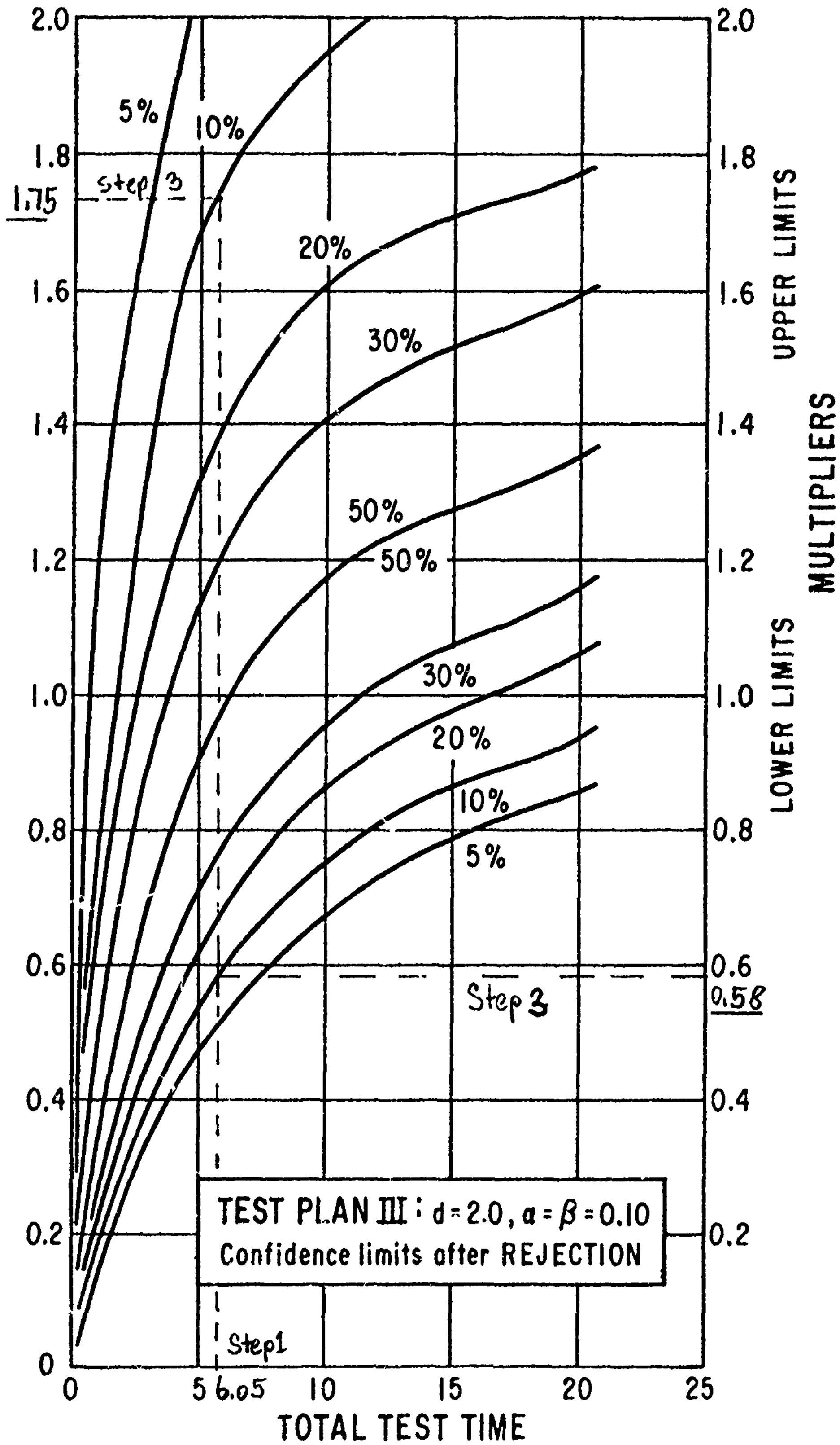


Figure 1b: Example of Steps in Obtaining Confidence Limits After Rejection

interval, but the upper test MTBF Θ_0 = 100 hours is not.

Other Charts and Tables

The charts cover all standard test plans of MIL-STD-781C. The acceptance charts are Figure 3, a-f. The rejection charts are Figure 4, a-h. Acceptance multipliers for Test Plans VIC and VIIIC are given in Table 2.

In calculating confidence intervals for these charts one follows the same steps as outlined in the previous example for Test Plan IIIC.

CONCLUDING REMARKS

1. Choice of the Confidence Level: Certain confidence levels seem to be more appropriate than others. The example used for confidence intervals after acceptance in test plan IIIC illustrates this. The test was terminated after not more than six failures occurred in 636 hours. The 80% two-sided confidence interval was calculated from 54 to 168 hours. Suppose one would have chosen the 90% confidence level instead. Following the steps as outlined in that section, the multipliers are 0.94 and 4.0 resulting in a 90% two-sided confidence interval from 47 hours to 200 hours. This interval includes both the lower test MTBF Θ_1 and the upper test MTBF Θ_0.

   This example shows that a confidence level above $(1-2\alpha)$ 100% for two-sided intervals and above $(1-\alpha)$ 100% for one-sided intervals (assuming $\alpha=\beta$) may result in intervals which include both Θ_0 and Θ_1.

2. Length of Confidence Intervals: As mentioned before, Harter's method usually results in shorter confidence intervals after acceptance than Epstein's.

   A similar comparison between intervals after a fixed-length test and a sequential test is more difficult, because the stopping rules are different. Equal number of failures in the same length of time usually do not occur.

Using an example from before shows this. A fixed-length test resulted in acceptance with seven failures after 940 hours. The 80% two-sided confidence interval on the MTBF is from 80 to 241 hours for Epstein's method, 84 to 220 hours for Harter's method. A sequential test with seven failures would have been terminated after only 705 hours with an 80% two-sided confidence interval from 53 to 152 hours. The interval after 705 hours of total sequential test time is only 99 hours long as opposed to 161 (or 136) hours after 940 hours of total fixed-size test time. However, the lower limit of the sequential interval is much closer to Θ_1 than the lower fixed-length limit. This is so, because the sequential test accepts (or rejects) as soon as possible. In other words, it accepts (or rejects) as soon as a $(1-2\alpha)$ 100% two-sided confidence interval is narrow enough <u>not</u> to cover both Θ_o and Θ_1.

REFERENCES

1. AROIAN, L.A. Application of the direct method in sequential analysis. Technometrics, 18, August 1976, pp 301-306.
2. AROIAN, L.A. and OKSOY, D. Estimation, confidence intervals, and incentive plans for sequential three-way decision procedures. 1972 NATO Conference Proceedings on Reliability Testing and Evaluation, VI-D-1 to FI-D-13.
3. BRYANT, C. and SCHMEE, J. Confidence Limits on MTBF for Sequential Test Plans of MIL-STD-781. Technometrics, 21, February 1979, to appear.
4. EPSTEIN, B. Estimation from life test data. Technometrics, 2, November 1960, pp 447-454.
5. FAIRBANKS, V.B. Two Sided Confidence Intervals for an Exponential Parameter. Technical Report No. 73, Dept. of Statistics, University of Missouri-Columbia, January 1978.
6. HAHN, G.J. Statistical Intervals for a Normal Population, Part I. Journal of Quality Technology, 2, July 1970
7. HAHN, G.J. Statistical Intervals for a Normal Population, Part II. Journal of Quality Technology, 2, October 1970, pp 195-206.
8. HARTER, H.L. New tables of the incomplete gamma function ratio and of percentage points of the chi-square ard beta distribution. U.S. Government Printing Office, Washington, D.C., 1964.

9. HARTER, H.L. MTBF Confidence Bounds Based on MIL-STD-781C Fixed-Length Test Results. Journal of Quality Technology, 10, October 1978, pp 164-169.

10. HARTER, H.L. and MOORE, A.H. An evaluation of exponential and Weibull test plans. IEEE Transactions on Reliability, R-25, June 1976, pp 100-104.

11. LUETJEN, P. Tables of parametric confidence limits from hypothesis test data. NAVSEC Report 6112-75-1, Naval Ship Engineering Center, Hyattsville, Maryland, 1974.

12. MIL-STD 781C, Military Standard Reliability Qualification on Production Acceptance Tests: Exponential Distribution, Washington, D.C., 1977.

13. NEATHAMMER, R.D., PABST, W.R. and WIGGINTON, C.G. MIL-STD 781B, reliability tests: exponential distribution. J. Quality Technology, 1, January 1965, pp 58-67.

14. SUMERLIN, W.T. Confidence calculations for MIL-STD 781. 1972 Annual Reliability and Maintainability Symposium. IEEE Catalog Number 72CH0577-B, January 1972, pp 205-212.

15. YASUDA, J. Correlation Between Laboratory Test and Field Part Failure Rates. IEEE Transactions on Reliability, R-26, June 1977, pp. 82-84.

Table 1. Accept-reject criteria for Test Plan IIIC

Number of Failures i	Total Test Time* Reject (Equal or Less) t_{Ri}	Accept (Equal or More) t_{Ai}
0	N/A	4.40
1	N/A	5.79
2	N/A	7.18
3	.70	8.56
4	2.08	9.94
5	3.48	11.34
6	4.86	12.72
7	6.24	14.10
8	7.63	15.49
9	9.02	16.88
10	10.40	18.26
11	11.79	19.65
12	13.18	20.60
13	14.56	20.60
14	15.94	20.60
15	17.34	20.60
16	20.60	N/A

* Total test time is total hours of equipment on time and is expressed in multiples of the lower test MTBF. Refer to 4.5.2.4 for minimum test time per equipment.

Table 2: Confidence Limits After Acceptance

Total Number of Failures	Multipliers							
	Lower Limits				Upper Limits			
	5%	10%	20%	30%	30%	20%	10%	5%
Test Plan VI: $d = 3.0$, $\alpha = \beta = 0.20$								
0	0.89	1.16	1.66	2.22	∞	∞	∞	∞
1	0.80	0.98	1.29	1.60	7.49	11.97	25.34	52.05
2	0.68	0.81	1.01	1.20	3.60	4.81	7.47	11.20
Test Plan VIII: $d = 2.0$, $\alpha = \beta = 0.30$								
0	0.57	0.75	1.069	1.43	∞	∞	∞	∞
1	0.54	0.66	0.88	1.09	4.82	7.71	16.32	33.53
2	0.53	0.65	0.83	1.00	2.49	3.33	5.18	7.77

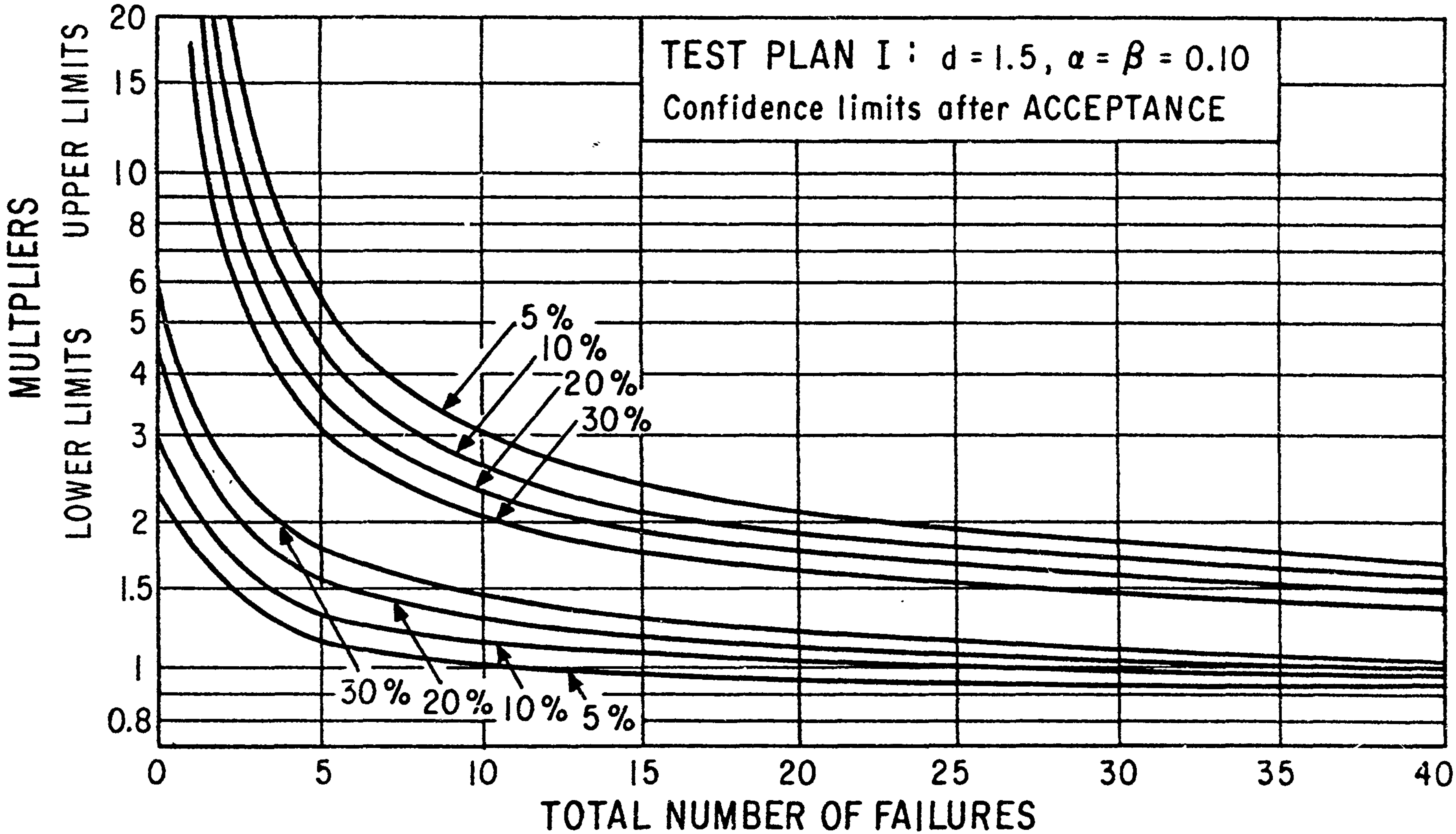


Figure 3a: Acceptance Chart for Test Plan Ic

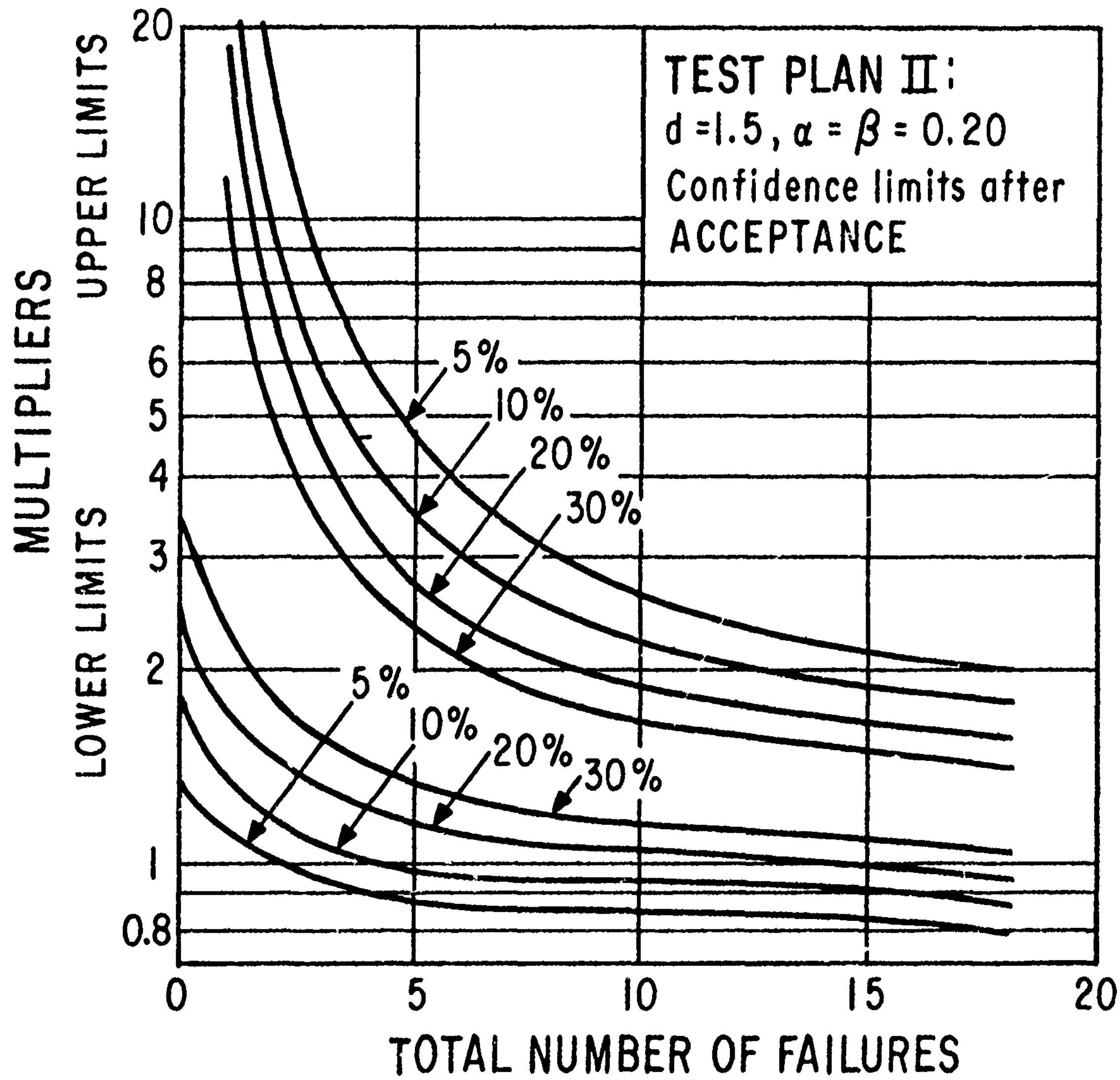


Figure 3b: Acceptance Chart for Test Plan IIc

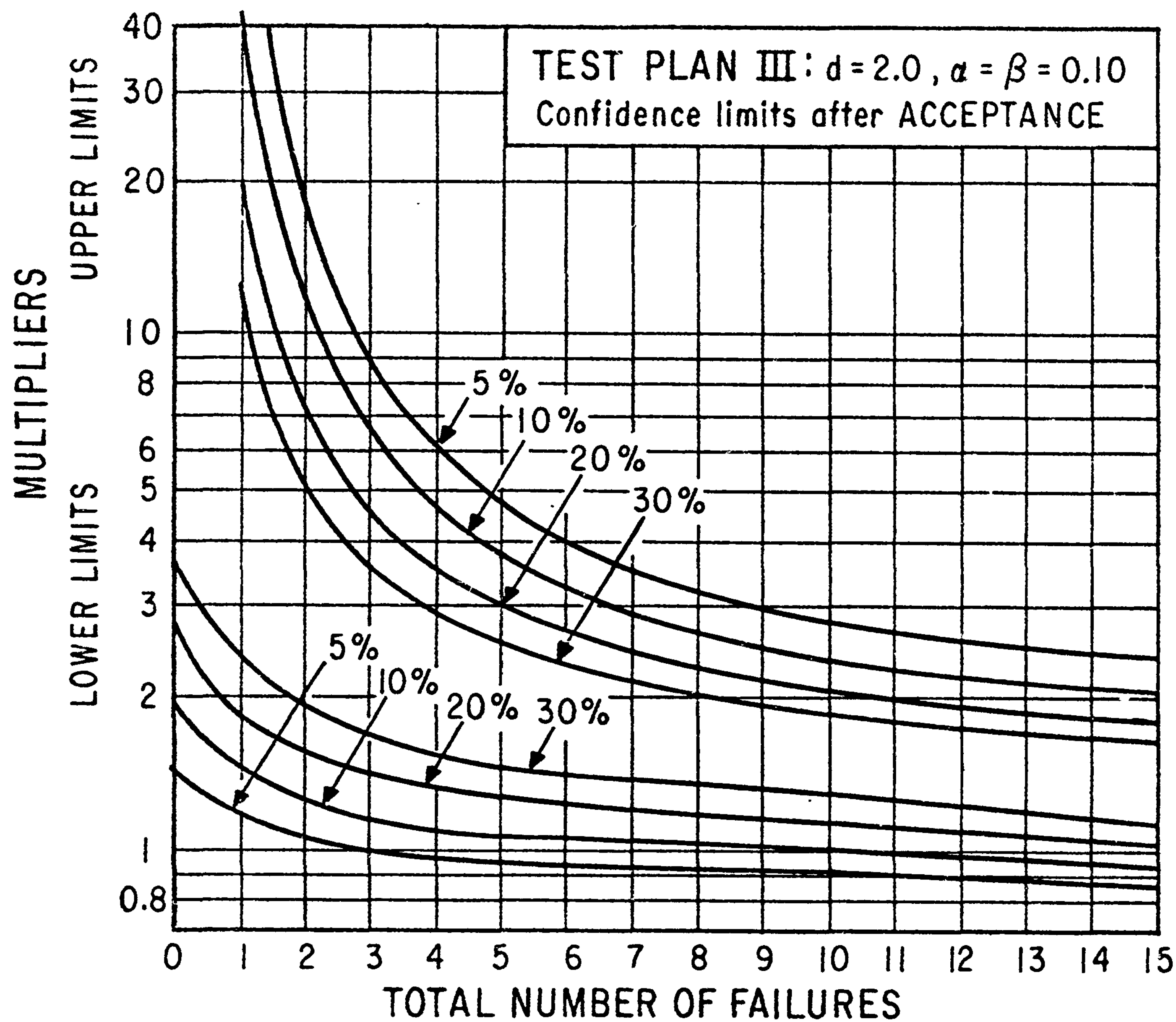


Figure 3c: Acceptance Chart for Test Plan IIIC

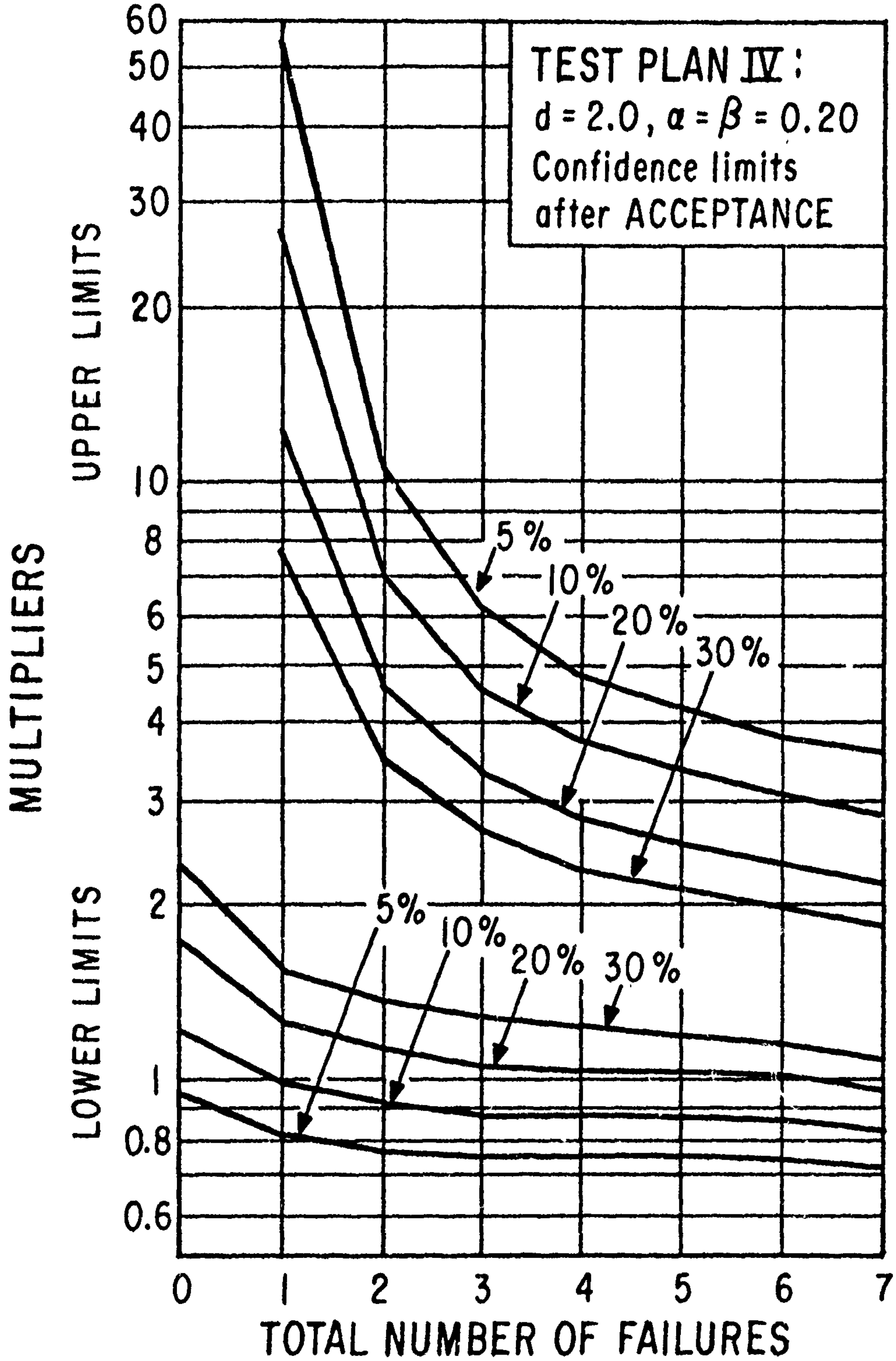


Figure 3d: Acceptance Charts for T.P. IVC

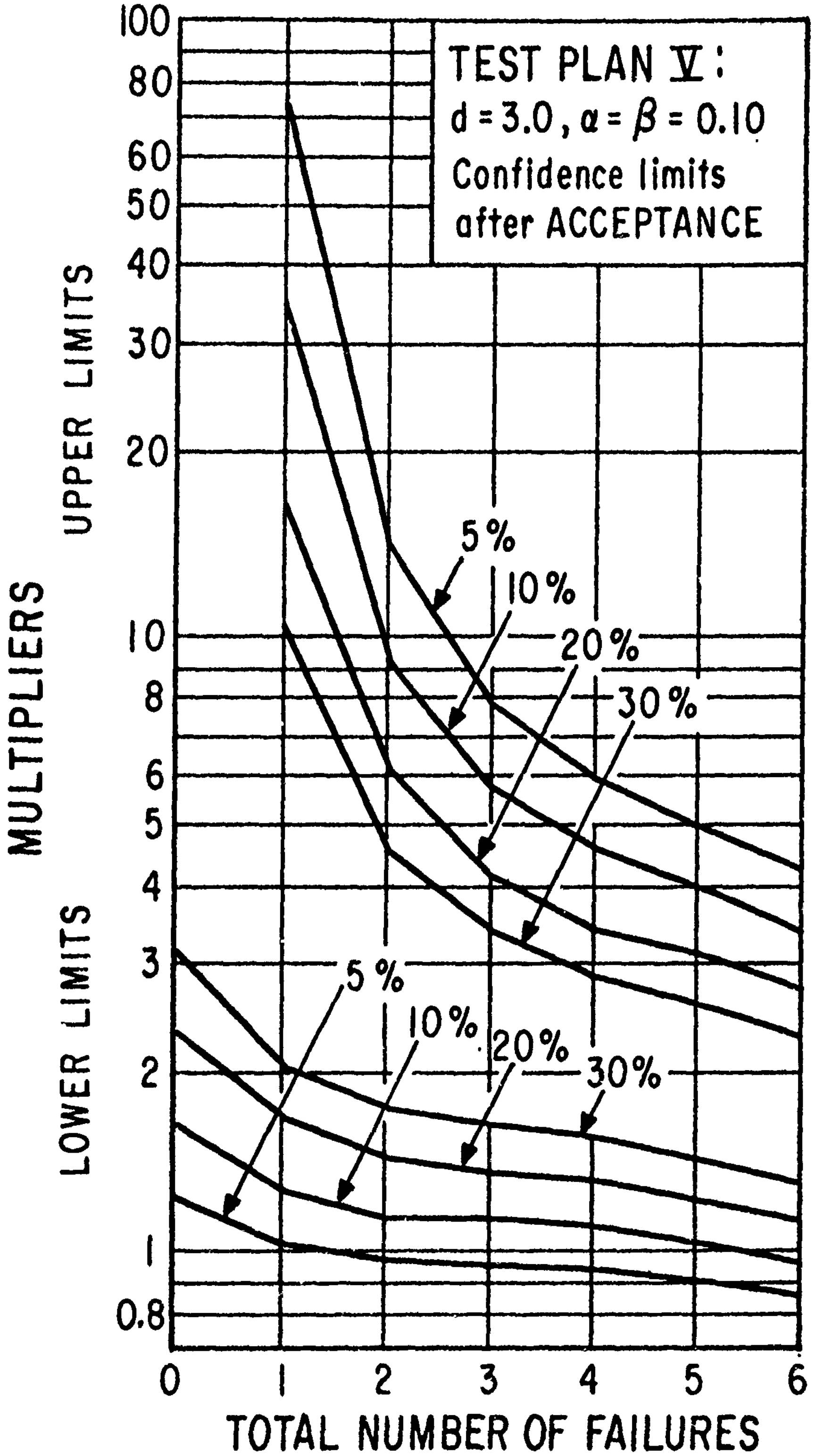


Figure 3e: Acceptance Chart for Test Plan VC

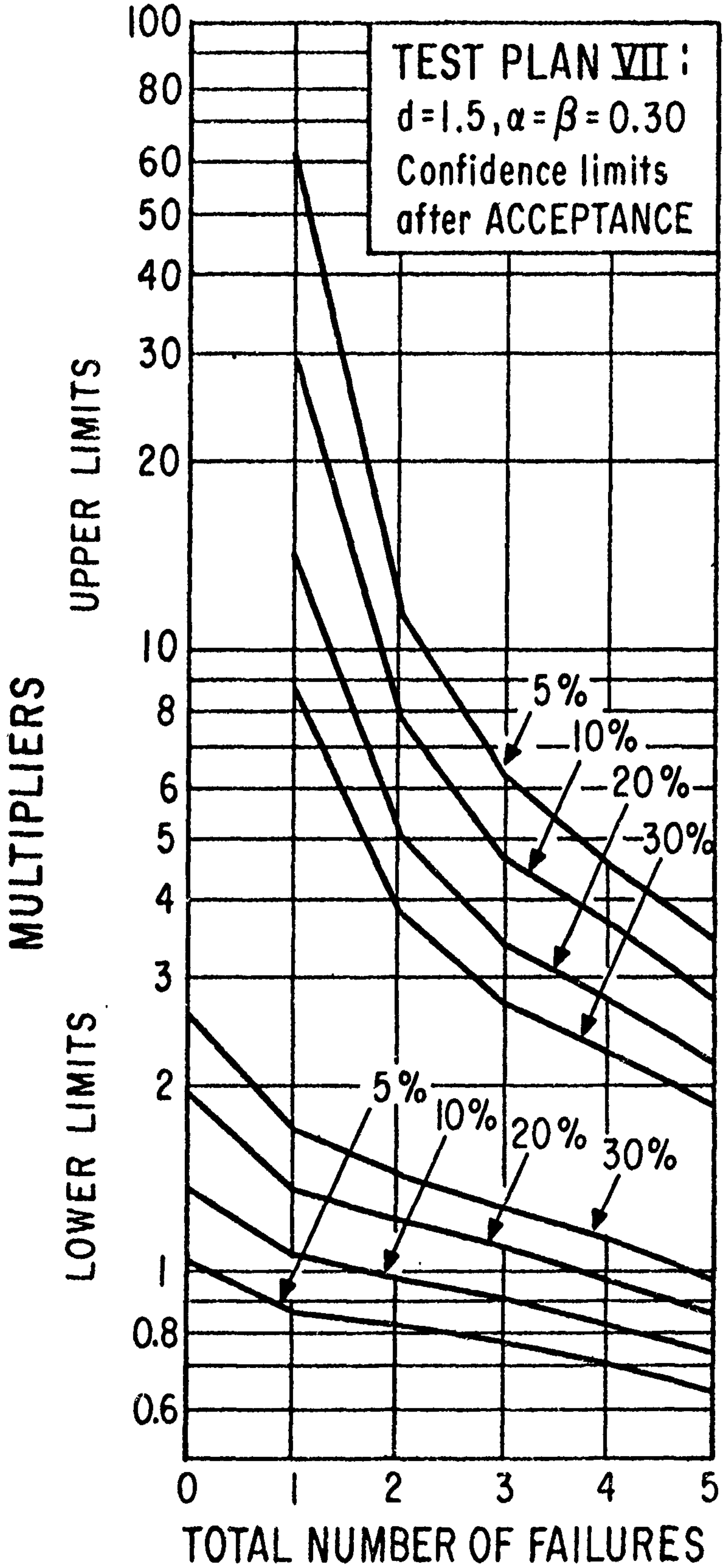
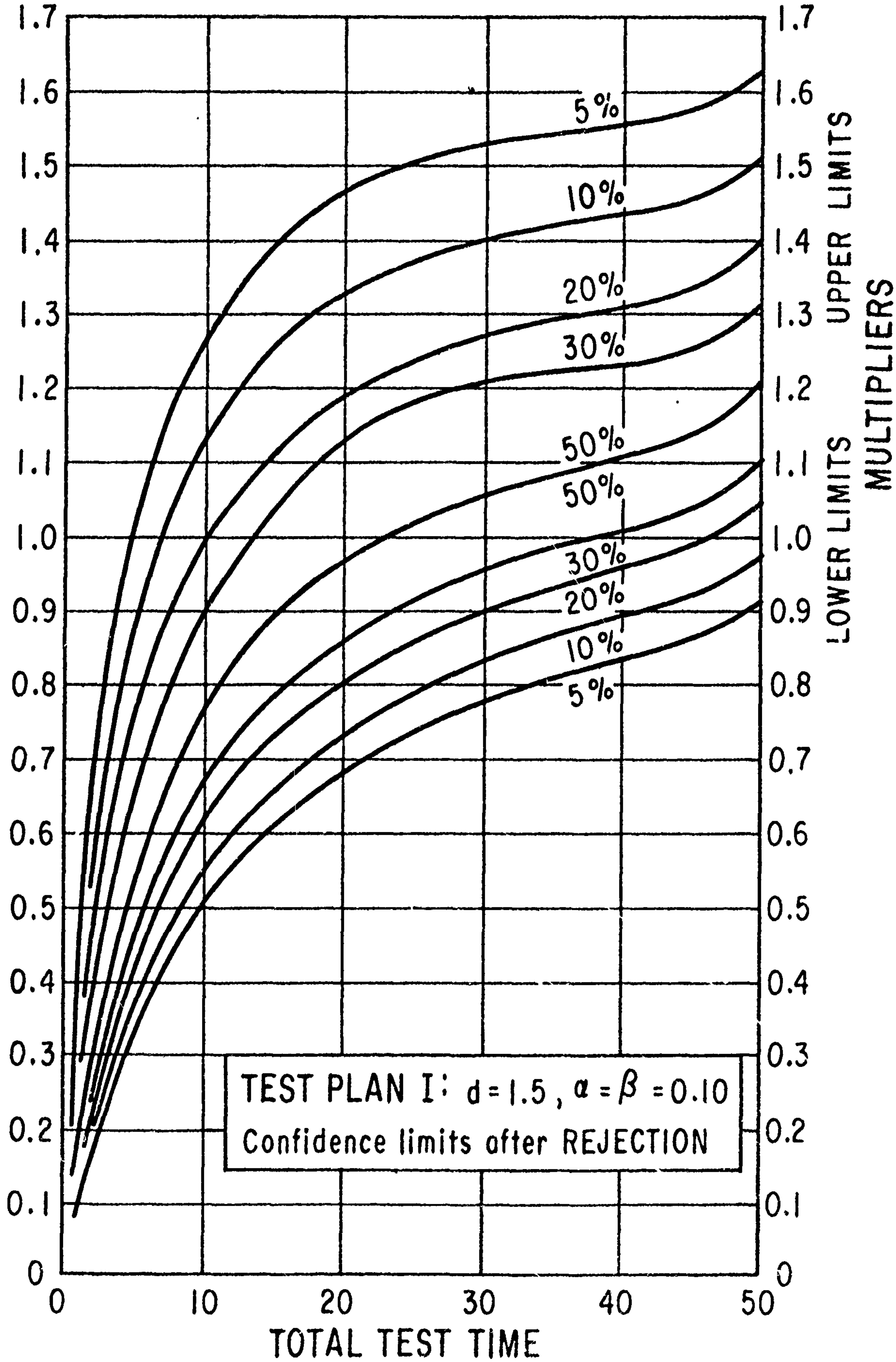
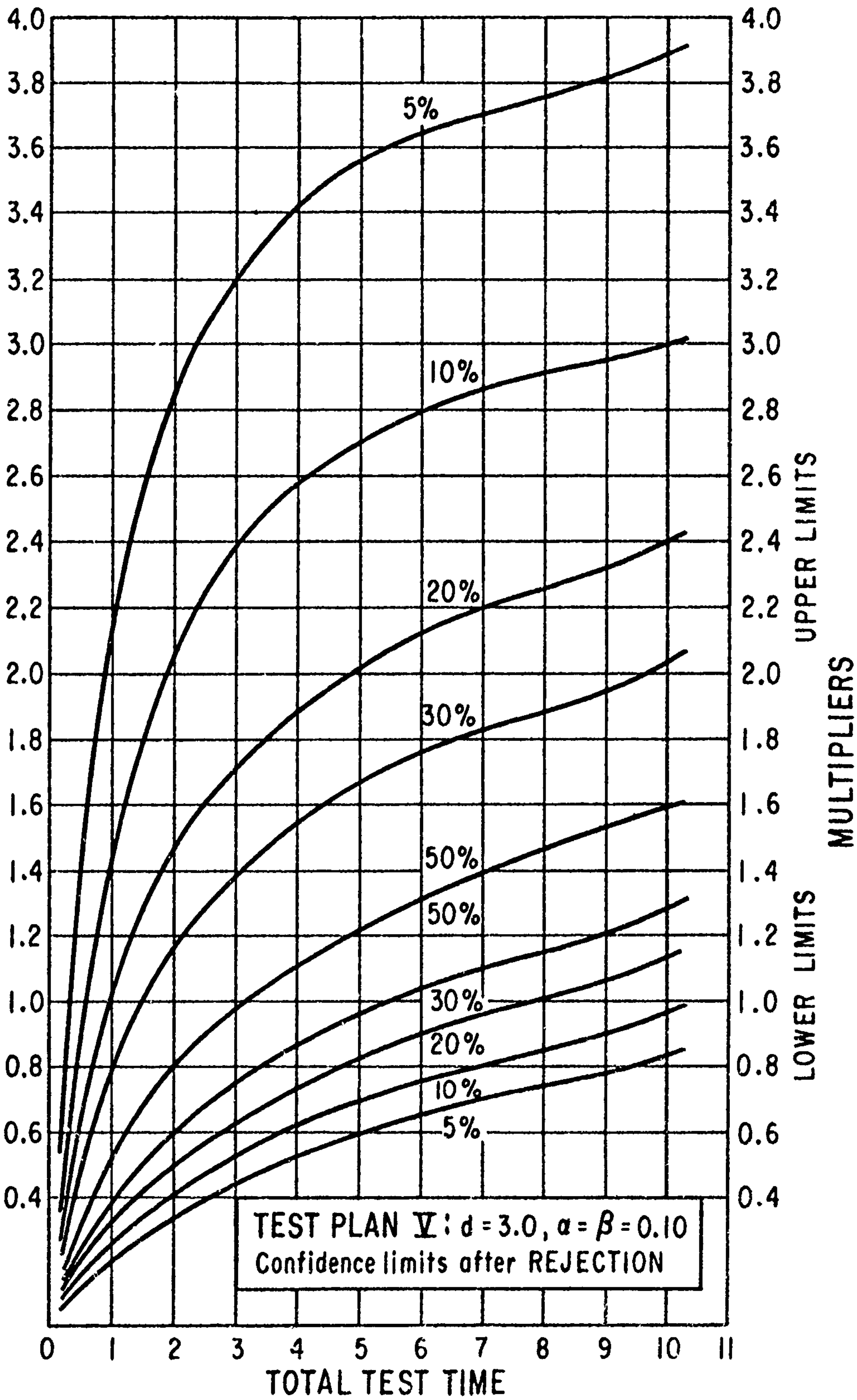


Figure 3f: Acceptance Chart for Test Plan VIIC



Fiaure 4a: Rejection Chart for Test Plan IC



Fiqure 4e: Rejection Charts for Test Plan VC

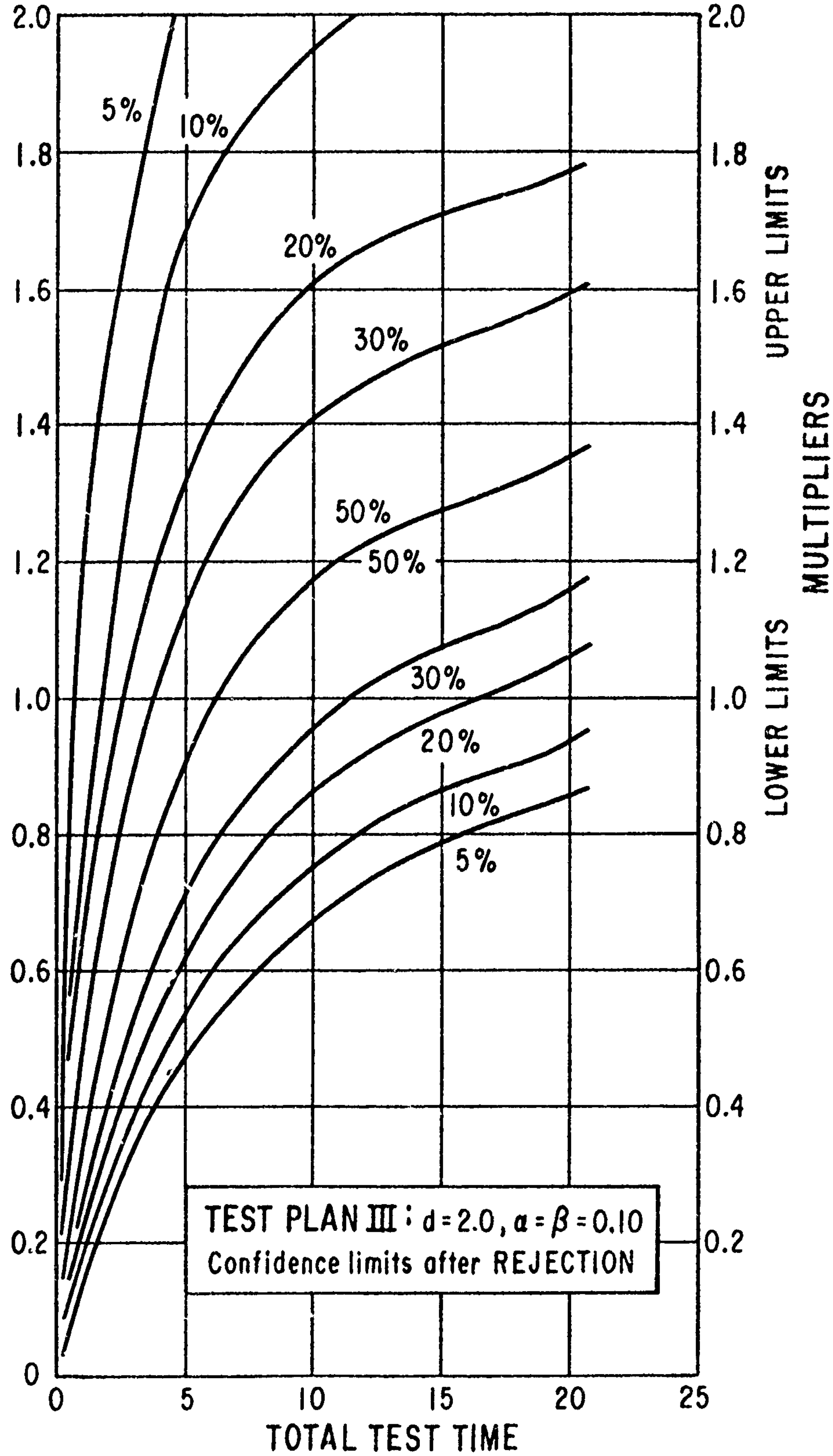


Figure 4C: Rejection Chart for Test Plan IIIC

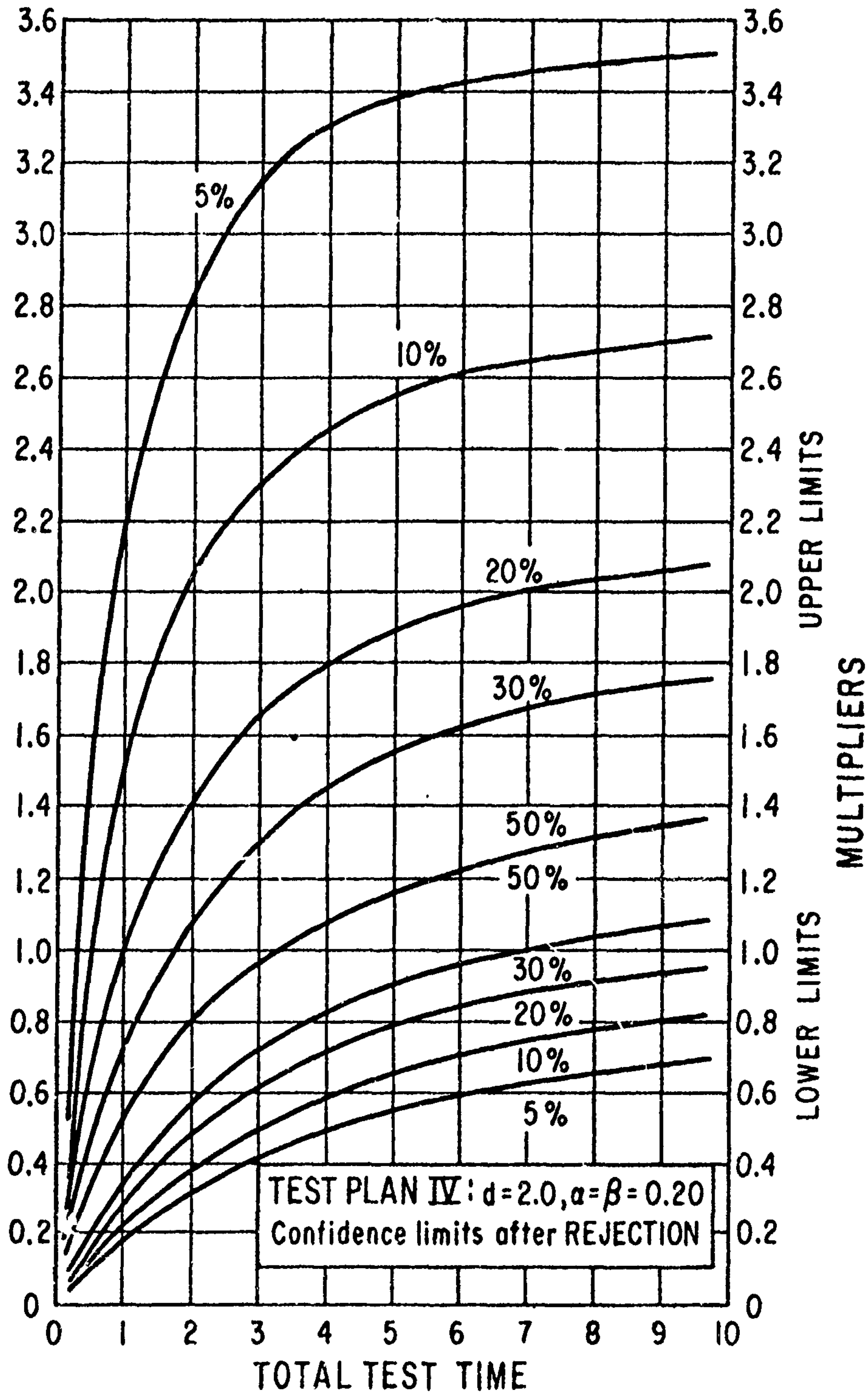


Figure 4d: Rejection Charts for Test Plan IVC

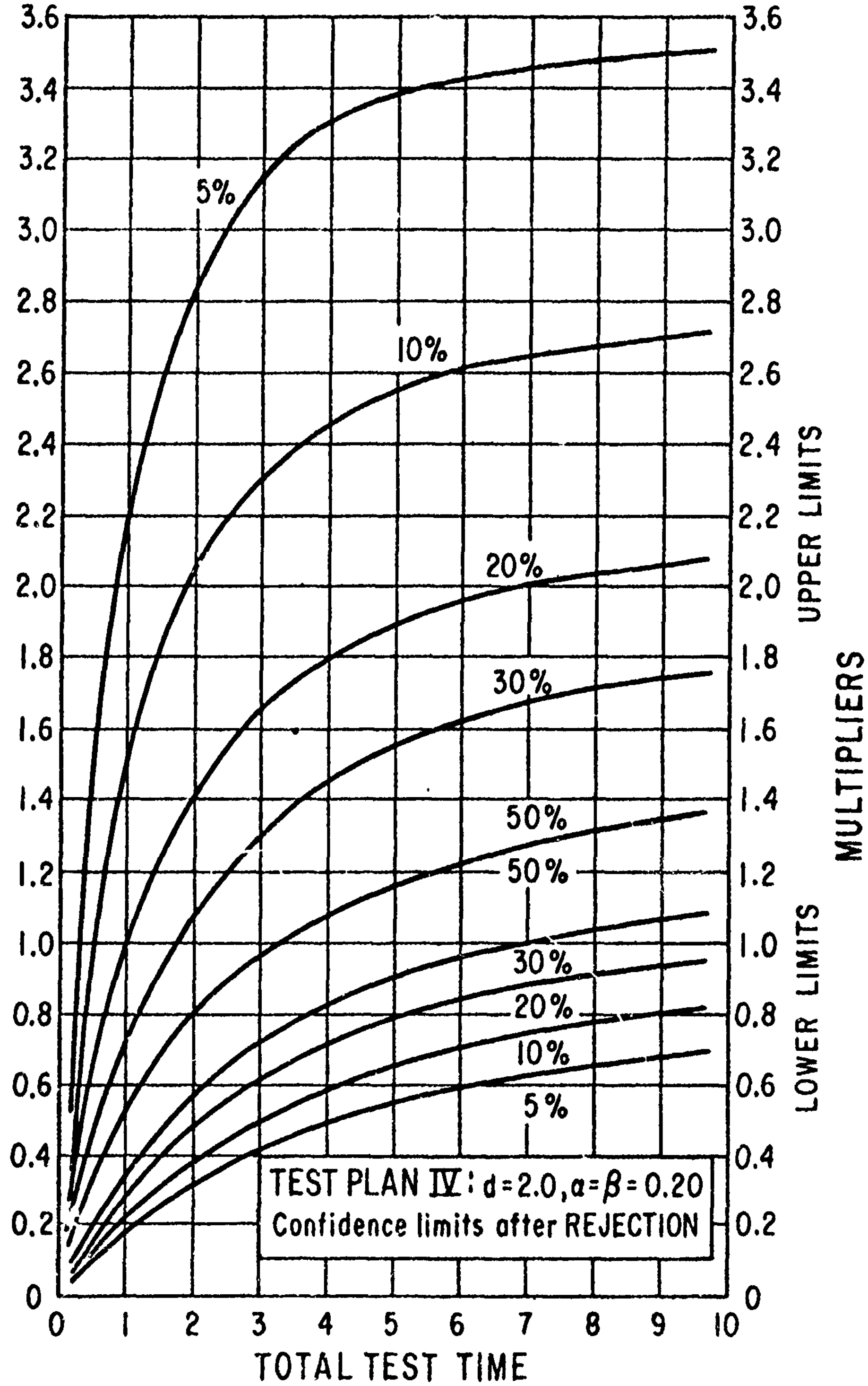


Figure 4d: Rejection Charts for Test Plan IVC

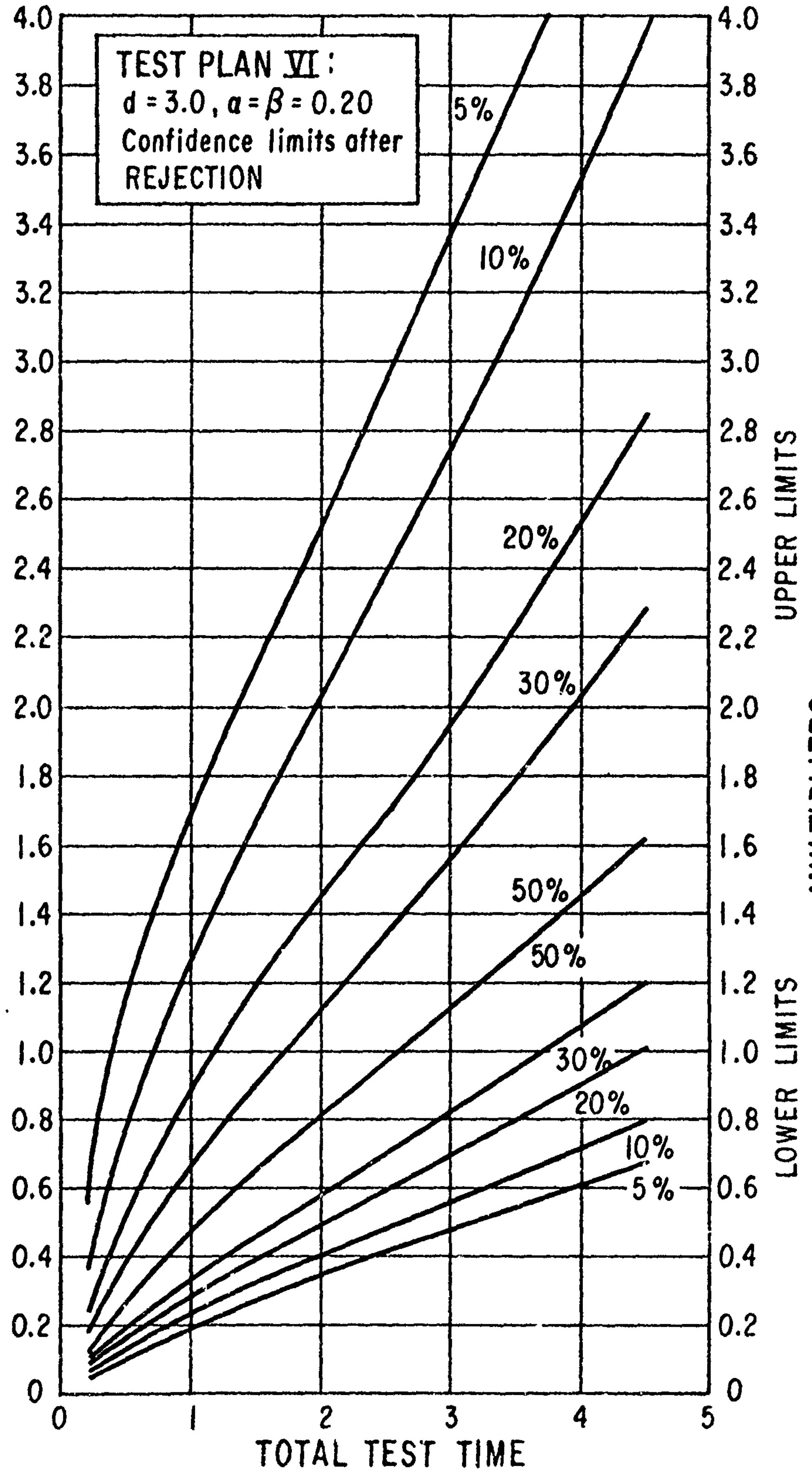


Figure 4f: Rejection Chart for Test Plan VIC

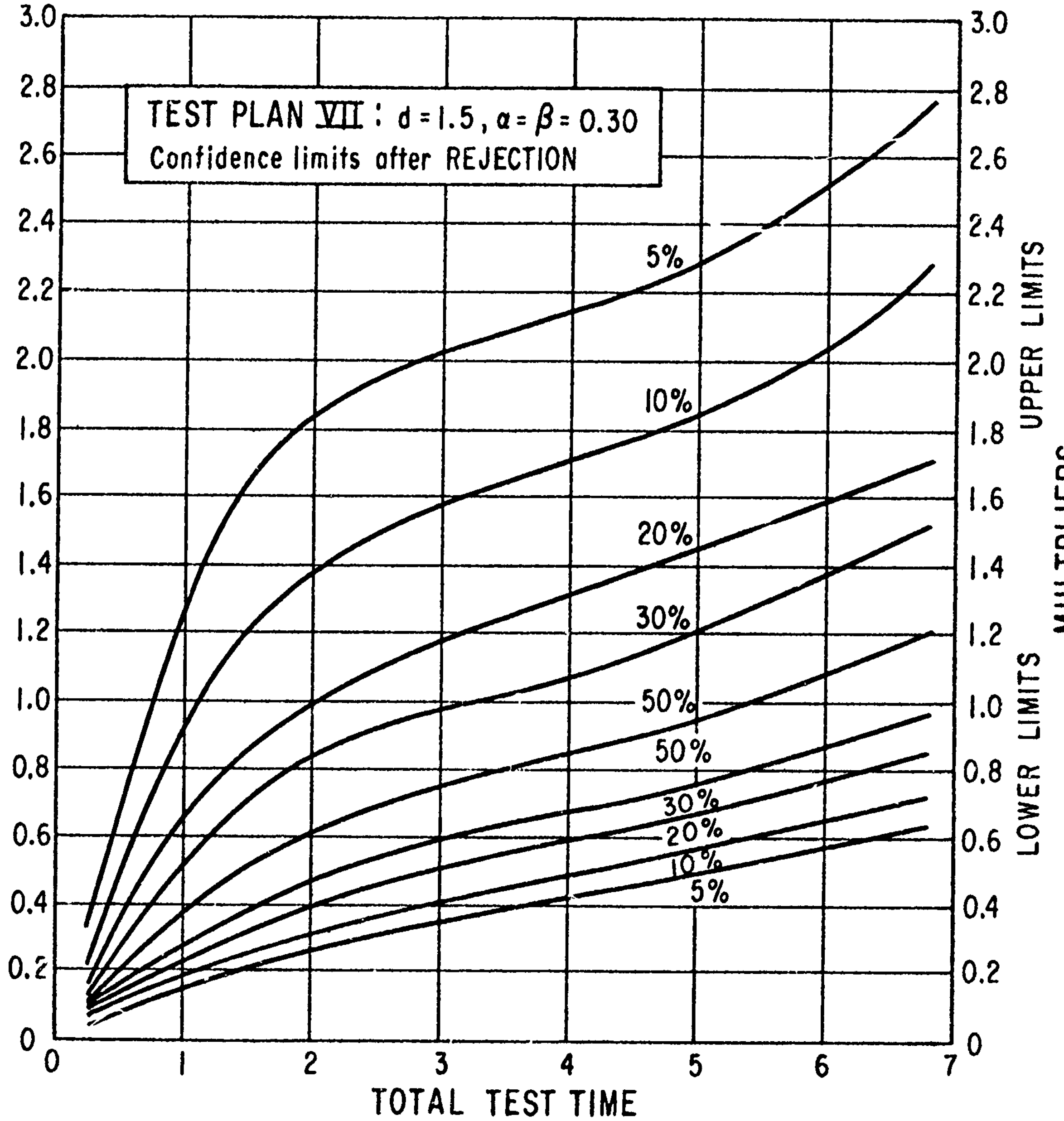


Figure 4q: Rejection Chart for Test Plan VIIC

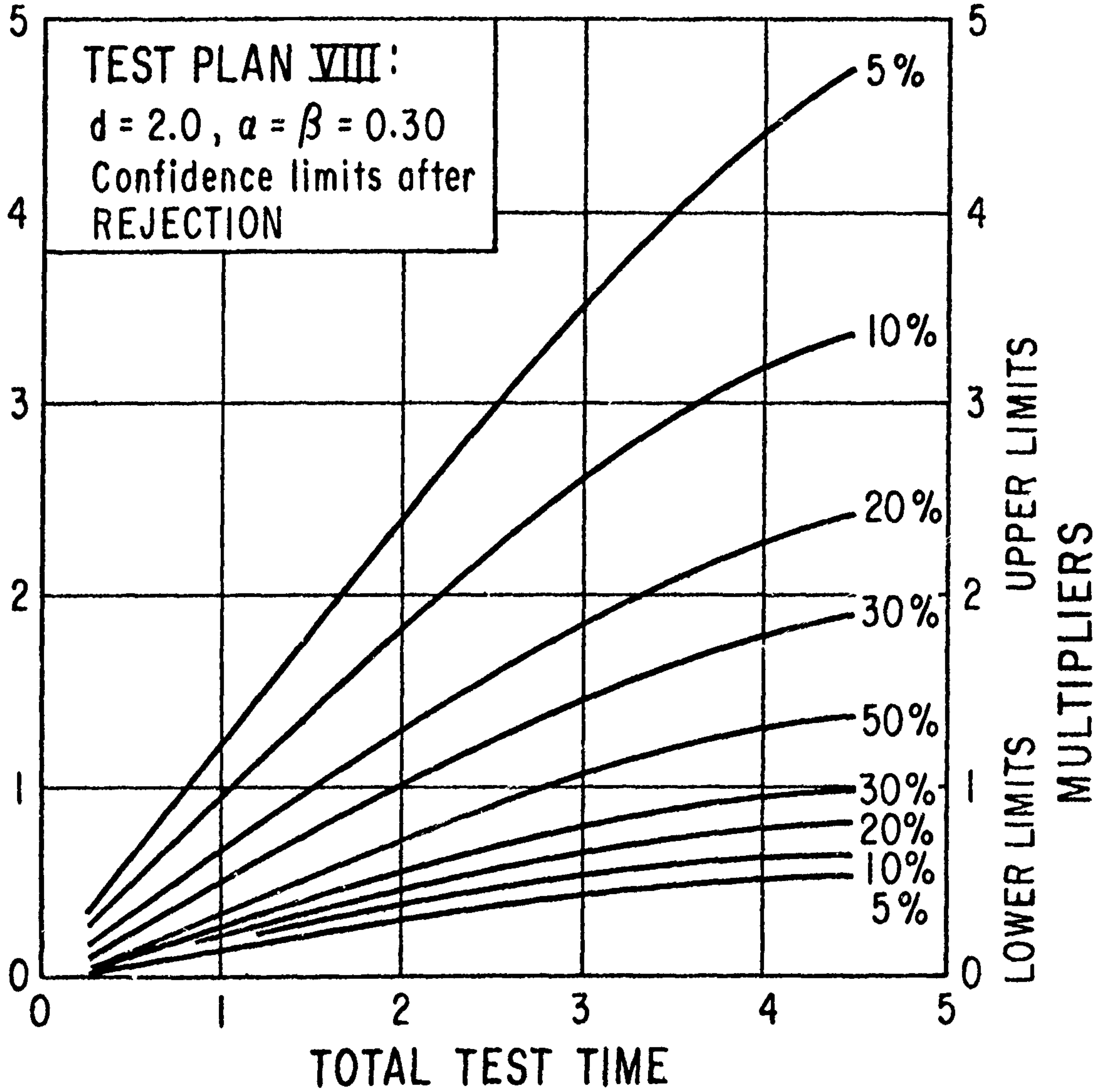


Figure 4h: Rejection Chart for Test Plan VIIIC

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AES-7905	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MIL-STD-781C and Confidence Intervals on Mean Time Between Failures		5. TYPE OF REPORT & PERIOD COVERED Technical Report-- 6/1/78-3/1/79
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Josef Schmee		8. CONTRACT OR GRANT NUMBER(s) N00014-77-C-0438
9. PERFORMING ORGANIZATION NAME AND ADDRESS Institute of Administration and Management, Union College Schenectady, NY 12308		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Department of Navy Arlington, VA		12. REPORT DATE Feb. 28, 1979
		13. NUMBER OF PAGES 25 + 14 charts & 2 tables
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Exponential distribution, confidence limits, sequential tests, fixed-length test, MTBF.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Various realistic examples illustrate how to obtain confidence limits on the mean time between failures (MTBF) of an exponential distribution from data obtained from one of the fixed-size or sequential test plans of MIL-STD-871C. For fixed-length tests, the methods developed by B. Epstein and the modifications of H.L. Harter are briefly discussed. For the sequential tests simple charts for newly developed methods of Bryant and Schmee are given.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)